Horizontal Symmetry Breaking in Jet-Induced 2-D Granular Crater Formation

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Problem Statement
We wish to understand crater formation in a 2-D granular bed driven by a jet of impinging gas, with potential applications to retrograde rocket landing on a planetary surface. We observe an unexpected breaking of horizontal symmetry, and we wish to characterize and explain this process, at least on a phenomenological level.

The Basics: Symmetric Digging
Mechanism: Impinging jet creates a high-pressure region directly below the nozzle. Gas flows out either side, sweeping a thin layer of particles with it. There is minimal penetration of gas into granular layer (at least in this regime).

Depth: logarithmic growth,
\[ D(t) = a \ln(bt + 1) \]
where: \( a \) = characteristic depth
\( 1/b \) = characteristic time
diggs out symmetrically.
Not unbounded, approaches asymptotic depth
Scalings:
\( a \) = height of jet (geometrical)
\( b \) = kinetic energy density of gas

Analysis: Differential Equations
Examination of the functional form of the depth evolution yields some fairly simple differential equations:
Erosion rate has exponential dependence on current depth
\[ \frac{d}{dt} \left( \frac{D(t)}{a} \right) = b \exp\left( -\frac{D(t)}{a} \right) \]
Fractional loss of erosion power is equal to the change in depth, scaled by the geometrical constant, \( a \), which is set by the jet height (sets crater’s shape)
This final equation is fairly intuitive, could be the basis of a simple phenomenological model of cratering process. Depth scale is set by crater geometry, time scale is set by kinetic energy of the impinging jet.

Horizontal Symmetry Breaking
At certain states, the cratering process will become asymmetric. Instead of digging symmetrically out of both sides, crater shifts positions so jet flows down one side of crater and out the other side (chooses randomly).

Questions:
1) Classification: Are there different types of symmetry breaking? Here, identified as “weak” and “strong”, based on behavior
2) Evolution: How does the system evolve as it undergoes this transition
3) Mechanism: Why does the symmetry break?

Weak Symmetry Breaking
Characteristics: Broader crater, debris is ejected mostly outward from the crater, little or no coupling between the different sides of the crater. Transitions are slower and, therefore, more easily controlled.

Evolution: Logarithmic depth growth not significantly disrupted. Horizontal evolution is quasi-linear.

Strong Symmetry Breaking (Continued)
Evolution: Logarithmic depth growth is significantly disrupted, and horizontal evolution is quasi-logarithmic (fitted with same log fit at depth). Data is plotted below in pairs with same initial conditions. Left pair broke at same time, right pair broke at different times. Notice the discontinuity of the horizontal positions at the transition point.

Possible Mechanism
Positive feedback for small fluctuations (crater shape): When the crater becomes slightly off-center due to fluctuations, the crater’s shape tends to direct the gas out the end that is farther away (see picture to the right).
Loss of stability from weakening jet as crater grows: At first, gas penetration into grains is more substantial. The rate of digging directly below the jet (centralized) is great enough to “stabilize” the crater’s position. As crater gets deeper, jet slows and spreads, less stable.
Results in efficient, lower resistance flow pattern: No high pressure region directly below the nozzle.

Hysteresis in Symmetry Breaking
The exact nature of this bifurcation is very complex. However, by running the system in an adiabatic, steady-state configuration, where the pressure is slowly varied and the system approximately stays at its asymptotic state, one can use pressure as a bifurcation parameter and clearly observe the hysteresis which is characteristic of a backward pitchfork bifurcation.

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