The Anatomy of Jam in a Two Dimensional Hopper

Junyao Tang and Robert P. Behringer
Department of Physics, Duke University
Durham, North Carolina, United States

Abstract
We seek an understanding of the physics of jamming for hopper flow using high speed spatio-temporal video data for photoelastic disks flowing through a two-dimensional hopper. We have found experimental support for the hypothesis that jamming events of granular flow in a hopper is approximately a Poisson process. The mean flow time between two consecutive jams increases rapidly with the hopper opening size, but it is insensitive to changes of the hopper wall angle.

Current work is focusing on investigating fluctuations of density/stress fields and their mutual correlations. These data are part of an IFPRI-NSF Collaborator for comparing physical data and simulations.

A Probability Hypothesis:
\[ P(t) = e^{-t/\tau} \]
Beverloo Equation:
Free Fall Arch Theory
\[ v = \frac{(g D)^{3/2}}{A - (D - h)^{1/2}} \]
\[ W = \frac{1}{A} (D - h)^{1/2} \]
Absence of Spatial Correlation

Therefore, Poisson Process, Survival Time Probability Distribution:
\[ P(t) = e^{-t/\tau} \] with \( P(0) = 1 \)

Results of \( \tau \)
A typical data sample:
\[ \tau = 11.5033 \text{ seconds} \]
\[ D = 3.3 \text{ cm} \]

Hypothesis of Probability Distribution of Survival Time Confirmed: \( P(t) = e^{-t/\tau} \)

Dependence of \( \tau \) on Hopper Opening and Angle

- Smaller Opening, Larger Average Stress and larger fluctuation
- Reason? Stronger spatial correlation of particles, more frequent intermittent jamming events

Anticorrelation between Stress/Velcity

- Quantitative proof of anti-correlation between stress and velocity: Stronger compressive stress fluctuation is the reason for more frequent intermittent jamming events

Insights from Mean Density Field

- Density falls towards opening
- Mean density profile little variation, weak impact on jamming probability

Insights from Density Fluctuation

- Reaching a high packing fraction is a necessary condition for jamming, but not sufficient.
- Other conditions that may affect: stability of an arch

Acknowledgement
This work has been supported by International Fine Particle Institute (IFPRI). We also appreciate many helpful discussions with Dr. Paul Mort of Proctor&Gamble.

Conclusions & Future Work

- Probability distribution of survival time of granular flow in a hopper exhibits an exponential form, indicates a mechanism of absence of spatial correlation of granular flow between low and high regions in a hopper.
- Mean survival time \( \tau \) grows exponentially with opening size, but depends weakly on hopper wall angle.
- Reaching a high packing fraction is a necessary condition for jamming, but not sufficient. Density fields have weak impacts on jamming mechanism.
- Stress fluctuation and localized force balance are more crucial for jamming